Math 115 – 09 (Calculus I)

University of Pittsburgh

Fall 2008

1. Section 9.5

Problem 2

The line through $(1, 0, -3)$ and parallel to $< 2, -4, 5 >$ has vector equation

$$\overrightarrow{r} =< 1, 0, -3 > +t < 2, -4, 5 >=< 1 + 2t, -4t, -3 + 5t >$$

and parametric equations

$$x = 1 + 2t, y = -4t, z = -3 + 5t.$$

Problem 8

$\overrightarrow{r_0} =< 2, 1, 0 >$ and parallel to $< a, b, c >$. Thus $< a, b, c >$ is perpendicular to $< 1, 1, 0 >$ and $< 0, 1, 1 >$. Since the dot product of perpendicular vectors is 0, we $a + b = 0$ and $b + c = 0$. Let $c = s$ (a parameter), then $b = -s$ and $a = s$. Any value will do, but let $s = 1$. Then the line is parallel to $< 1, -1, 1 >$. So a parametric equation is

$$\overrightarrow{r} =< 2, 1, 0 > +t < 1, -1, 1 >=< 2 + t, 1 - t, t >$$

and possible symmetric equations are

$$\frac{x - 2}{1} = \frac{y - 1}{-1} = \frac{z - 0}{1}$$

which simplifies to

$$x - 2 = 1 - y = z.$$

Problem 18

Not parallel b/c their components are not proportional.

If they intersect, then

$$1 + 2t = -1 + s \qquad (1)$$

$$3t = 4 + s \qquad (2)$$

$$2 - t = 1 + 3s \qquad (3)$$

Solving equations (1) and (2): $t = 6, s = 14$ Solving equations (1) and (3): $t = -\frac{5}{7}$ and who cares about s. Since the solutions do not agree, the lines do not intersect. Hence they are skew.

Problem 22
Scalar equation is $0(x-4)+1(y-0)+2(z--3)=0$ which simplifies to $y+2z+6=0$.

2. SECTION 9.6

Problem 6
$f(x,y)=\sqrt{xy}$ hence the domain is $\{(x,y)|xy>0\}$. $xy>0$ iff $x>0, y>0$ or $x<0, y<0$ thus the graph is the first and third quadrants.

3. SECTION H.1

Problem 2
a) $(3,0)$ in polar coordinates is $(3,2\pi)$ or $(-3,\pi)$ (note: any coterminal angle would work; so the θ's may vary by $\pm 2\pi$).

b) $(2,-\frac{\pi}{7})$ in polar coordinates is $(2,\frac{13\pi}{7})$ or $(-2,-\frac{\pi}{7})$ (note: any coterminal angle would work; so the θ's may vary by $\pm 2\pi$).

c) $(-1,-\frac{\pi}{2})$ in polar coordinates is $(1,\frac{\pi}{2})$ or $(-1,\frac{3\pi}{2})$ (note: any coterminal angle would work; so the θ's may vary by $\pm 2\pi$).

Problem 4
a) $(2,\frac{2\pi}{3})$ is $(-1,\sqrt{3})$.

b) $(4,3\pi)$ is $(-4,0)$.

c) $(-2,-\frac{5\pi}{6})$ is $(\sqrt{3},1)$.

Problem 16
Since $\tan\theta=\frac{y}{x}$ and $\sec\theta=\frac{r}{x}$,

$$r=\tan\theta\sec\theta=\frac{y}{x}\cdot\frac{r}{x}.$$

Multiplying both sides by $\frac{x^2}{r}$ gives $y=x^2$ where $x\neq 0+2k\pi$ (else $\tan\theta,\sec\theta$ are undefined).

4. SECTION 9.7

Problem 8

a)

$$x = \rho\cos\theta\sin\phi = 5\cos\pi\sin\frac{\pi}{2} = -5$$
$$y = \rho\sin\theta\sin\phi = 5\sin\pi\sin\frac{\pi}{2} = 0$$
$$z = \rho\cos\phi = 5\cos\frac{\pi}{2} = 0$$

b)

$$x = \rho\cos\theta\sin\phi = 4\cos\frac{3\pi}{4}\sin\frac{\pi}{3} = -\sqrt{6}$$
$$y = \rho\sin\theta\sin\phi = 4\sin\frac{3\pi}{4}\sin\frac{\pi}{3} = \sqrt{6}$$
$$z = \rho\cos\phi = 4\cos\frac{\pi}{3} = 2$$

Problem 10

a)

$$\rho = \sqrt{0^2 + \sqrt{3}^2 + 1^2} = 2$$
$$\tan\phi = \frac{\sqrt{3}}{1}, \text{ so } \phi = \frac{\pi}{6}$$
$$\theta = \frac{\pi}{2} \text{ by inspection}$$

b)

$$\rho = \sqrt{(-1)^2 + 1^2 + \sqrt{6}^2} = 2\sqrt{2}, \text{ so}$$
$$\sqrt{6} = 2\sqrt{2}\cos\phi \Rightarrow \phi = \frac{\pi}{6} \text{ since } y > 0$$
$$1 = 2\sqrt{2}\sin\theta\sin\frac{\pi}{6} \Rightarrow \sin\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{4}$$