

NAME: _____

Math2701-001

Exam 1: Sections 1.1 – 2.2 (5th ed.); 1.1 – 3.2 (6th ed.)

Discrete Structures

Please show all work. When some work is required, **NO WORK = NO CREDIT.**

1. (4 points) Determine whether each of the following logic statements is true or false:
 - (a) $1 + 1 = 3$ if and only if today is Tuesday.
 - (b) If $5 - 3 = 0$, then Jeff is a complete moron.
2. (8 points) Show that $p \rightarrow q$ is equivalent to $\neg p \vee q$.

3. (4 points) Evaluate the following statement involving bit strings:

$$(01111 \wedge 10111) \oplus 10001.$$

4. (6 points) Determine the truth value of each of these statements if the universe of discourse of each variable consists of all real numbers:

(a) $\forall x \exists y (x^2 = y)$

(b) $\exists x \forall y (x^2 = y)$

(c) $\exists x \forall y (xy = 0)$

5. (3 points) Rewrite the following statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives):

$$\neg(\forall x \exists y [P(x, y) \vee Q(x, y)]).$$

6. (6 points) Prove that the product of two odd numbers is an odd number.

7. (10 points) Prove that the following are equivalent:

1) x is rational,

2) $\frac{1}{x}$ is rational,

3) $2x - 7$ is rational.

8. (12 points) Determine whether each of these statements are true or false:
- (a) $x \in \{x\}$
 - (b) $\{x\} \subseteq \{x\}$
 - (c) $\{x\} \in \{x\}$
 - (d) $\{x\} \in \{\{x\}\}$
 - (e) $\emptyset \subseteq \{x\}$
 - (f) $\emptyset \in \{x\}$
9. (10 points) What is the cardinality of each of these sets?:
- (a) \emptyset
 - (b) $\{\emptyset\}$
 - (c) $\{\emptyset, \{\emptyset\}\}$
 - (d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$
 - (e) $P\{\emptyset, \{\emptyset\}\}$
10. (8 points) If $A = \{a, b, c\}$ and $B = \{1, 2\}$. Find $A \times B$.

11. (8 points) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 1, 3, 6\}$. Find

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $B - A$

12. (6 points) Let A and B be sets. Show that $A \cup B = B \cup A$.

13. (4 points) The **symmetric difference** of A and B , denoted by $A \oplus B$, is the set containing those elements in A or B , but not in both A and B .
- (a) Find the symmetric difference of $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$.
- (b) Draw a Venn diagram for the symmetric difference of arbitrary sets C and D .
14. (5 points) Draw a graph of $f(x) = \lfloor \frac{x}{3} \rfloor$.
15. (6 points) Use the definition of $f(x)$ is $O(g(x))$ to show that $x^5 + 7x^3 + 12x + 99$ is $O(x^4)$.